A LOSSY CODING SCHEME FOR IMAGES BY USING THE HAAR WAVELET TRANSFORM AND THE THEORY OF IDEAL CROSS-POINTS REGIONS

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ABSTRACT

This paper presents Lossy Coding Scheme for Images by Using The Haar Wavelet Transform and The Theory of Cross-points Regions with Ideal Cross-points Regions (HWTICR). The base of this statement is the effect of Gray coding on cross-points which are neighbor to the points of grey levels 2^n. After Gray coding these regions always contain only 1-bits or 0-bits depending on the number of each bit plane after bit plane decomposition. The optimization of probability in each bit plane has important effects on encoding and decoding processes of lossless image compression for data transmission. The framework itself is founded upon a wavelet transformed domain, the scheme will show how The Haar Wavelet Transform combines with the theory of Ideal Cross-points Regions to become a lossy coding scheme for images. The goal of the method is to build a lossy coding scheme for images with high compression ratio and low distortion factor in comparison with some other methods. Finally, some initial results of the scheme are also presented and compared to the other methods. The algorithm can be used in medical and photographic imaging.

Keywords: cross-point, Gray code, ideal cross-point regions, quantization, Wavelets Transform, 2D - Discrete Haar Wavelet Transform.

1. INTRODUCTION

This paper is developed from paper [1, 2] that presented the definition of cross-points regions and ideal cross-points regions, the propositions on bit states of cross-points and the consequences from the entropy of obtained data on cross-points regions after Gray coding on bit planes n-1, and n-2 with n being the component of grey levels 2^n. The application of the ideal cross-points regions and the bit states [3], the characteristics of Gray code of cross-points and the scheme Jones for lossless compressed image data for building a lossless coding scheme for images.

In this paper, the 2D-Discrete Haar Wavelet Transform (DHWT) is used with the lossy coding scheme that averages some regions in the matrix result of the Haar Wavelet Transform.

The matrix of input images will be quantized before applying DHWT. After using DHWT with the lossy coding scheme, the lossy matrix will be compressed by using scheme Jones with the characteristics of Gray code in ideal cross-points regions.

This paper has six sections. After this introduction, Section 2 mentions the definition of ideal cross-points regions and the bit states, and propositions about characteristics of Gray codes of cross-points a part of which is presented in [4 - 6]. The combination of Haar wavelet transform and the theory of ideal cross-points regions is presented in Section 3. Section 4 introduces the lossy coding scheme using the Haar wavelet transform and the theory of ideal cross-points regions. In Section 5, some of our results obtained from using the above theory with Jones' algorithm [7] are presented and compression ratio in comparison with JPEG and JPEG2000. Section 6 contains the conclusion and the scope for future research.

2. IDEAL CROSS-POINTS REGION AND BIT STATES

2.1. Effect of Gray coding on data bits in ideal cross-points regions on the bit plane $n-1$

Definition 2.1. *Let the positive integer N be the bit length of data points, the region of cross-points A_0 (n), with n from 1 to (N − 1), is a set of data points whose grey values are from ($2^n - 2^{n-1}$) to ($2^n + 2^{n-1} - 1$). The point of grey value 2^n (if existing) is called the center point of the cross-points region, and the grey value 2^n is called the central value. These regions are called the ideal cross-points regions (ICRs) of type A.*

With Definition 2.1 data points in ICRs have grey values that satisfy the rule

$$V_A(n) = \{2^n - 2^{n-1}, ..., 2^n + 2^{n-1} - 1\}. \quad (1)$$

The value n is the exponent of central values 2^n, it is also used to determine the number of bit plane for data bit compression. The rule $V_A(n)$ can be divided into two groups : V_{Al} (n) and V_{Ag} (n) with

$$V_{Al}(n) = \{2^n - 2^{n-1}, 2^n - 2^{n-1} + 1, ..., 2^n - 2, 2^n - 1\},$$

$$V_{Ag}(n) = \{2^n, 2^n+1, ..., 2^n + 2^{n-1} - 2, 2^n + 2^{n-1} - 1\}.$$

For example, when n = 3, $V_A(n) = \{2^3 - 2^{3-1}, ..., 2^3 + 2^{3-1} - 1\} = \{4, 5, 6, 7, 8, 9, 10, 11\}$.

Proposition 2.1. *Let n be the exponent of the central value 2^n in the ideal cross-points region A_0 (n), n is from 1 to (N −1), N is a positive integer of bit length. Bits of Gray codes in the ideal cross-points region A_0 (n) on the bit plane n−1 are bits 1.*

Proof of Proposition 2.1. According to Definition 2.1, with a value of n in the interval 1÷(N−1), grey values of data points in the region $A_0(n)$ that satisfy $V_{A1}(n)$ are expanded under the form of polynomials of radix 2 as the following

$$0.2^{N-1} + ... + 0.2^n + 1.\ 2^{n-1} + x.\ 2^{n-2} + ... + x.\ 2^0. \quad (2)$$

Data points in the region $A_0(n)$ that satisfy V_{Ag} (n) are expanded by the following polynomial

$$0.2^{N-1} + ... + 0.2^{n+1} + 1.2^n + 0.2^{n-1} + x.2^{n-2} + ... + x\ 2^0, \quad (3)$$

where x are bits 1 or 0.

After Gray code transformation, (2) and (3) become (4) and (5) respectively

$$0.2^{N-1} + ... + 0.2^{n} + \mathbf{1.2^{n-1}} + x.2^{n-2} + ... + x.2^{0}, \quad (4)$$

$$0.2^{N-1} + ... + 0.2^{n+1} + 1.2^{n} + \mathbf{1.2^{n-1}} + x.2^{n-2} + ... + x.2^{0}. \quad (5)$$

By combining (4) and (5), the region $A_0(n)$ on the bit plane $n - 1$ always contains 1 bits.

For example, when n = 3, the central value is 23, $A_0(3)$ = {4, 5, 6, 7, 8, 9, 10, 11}. After Gray coding Gray codes of these values are 6, 7, 5, 4, 12, 13, 15, 14 respectively, all of them have 1-bits on the bit plane 2 (= 3 − 1). This is very good to compress data because the probability of 1-bit in ICRs is always 1, and the probability of 0-bit in ICRs is always 0. This problem plays an important role to optimize the probability of data bits in the step of modeling of entropy coding.

Consequence 2.1. *In the ideal cross-points regions A_0, after Gray code transformation the entropy of obtained data on a certain bit plane is minimum.*

Proof of Consequence 2.1. We can easily see that in the ideal cross-points regions $A_0(n)$ on the bit plane $n-1$, before Gray code transformation, the probabilities of 1-bit and 0-bit are random, therefore the entropy of the corresponding bit string is often H = −P (1).$\log_2$ P (1)− P (0). Log_2 P (0) > 0.

After Gray code transformation, the ideal cross-points regions contain all 1 bits, so the probability of 1-bit in these regions is 1, and the probability of 0-bit is 0 there. This is the reason for that the entropy of this bit string is always H = −P (1). Log_2 P (1)− P (0). Log_2 P (0) = 0, that means the average information of this region is 0.

2.2. Effect of Gray coding on data bits in ideal cross-points regions on the bit plane *n*−2

Definition 2.2. *Let the positive integer N be the bit length of data points, the region of cross-points R_0 (n), with n from 2 to (N − 1), is a set of data points whose grey values are from (2^n – 2^{n-2}) to (2^n + 2^{n-2}−1). The point of grey value 2^n (if existing) is called the center point of the cross-points region, and the grey value 2^n is called the central value. These regions are called the ideal cross-points regions (ICRs) of type R.*

According to the definition 2.2, grey values of data points in ICRs are in the following set

$$V_R(n) = \{ 2^{n} - 2^{n-2}, ..., 2^{n} + 2^{n-2} - 1\}. \quad (6)$$

The exponent n of central values 2^n is from 2 to (N − 1). The rule $V_R(n)$ includes two groups: $V_{Rl}(n)$ and $V_{Rg}(n)$ with

$$V_{Rl}(n) = \{2^{n} - 2^{n-2}, 2^{n} - 2^{n-2} + 1, ..., 2^{n} - 2, 2^{n} - 1\},$$
$$V_{Rg}(n) = \{2^{n}, 2^{n}+1, ..., 2^{n} + 2^{n-2} - 2, 2^{n} + 2^{n-2} - 1\}.$$

For example, when n = 3, $V_R(n) = \{2^3 - 2^{2-1}, ..., 2^3 + 2^{2-1} - 1\}$= {6, 7, 8, 9}.

Proposition 2.2. *Let n be the exponent of the central value 2^n in the ideal cross-points region $R_0(n)$, n is from 2 to (N −1), N is a positive integer of bit length. Bits of Gray codes in the ideal cross-points region $R_0(n)$ on the bit plane n−2 are bits 0.*

Proof of Proposition 2.2. Be based on Definition 2.2, with a value of n in the interval 2 ÷ (N −1) grey values of data points in the region $R_0(n)$ are from (2^n – 2^{n-2}) to (2^n – 2^{n-2}−1), so they may be expanded under the form of polynomials of radix 2(7) and/or (8) as the followings

$$0.2^{N\text{-}1}+...+0.2^{n}+1.2^{n\text{-}1}+1.2^{n\text{-}2}+x.2^{n\text{-}3}+...+x.2^{1}+x.2^{0}, \quad (7)$$

$$0.2^{N\text{-}1}+...+0.2^{n+1}+1.2^{n}+0.2^{n\text{-}1}+0.2^{n\text{-}2}+x.2^{n\text{-}3}+...+x.2^{0}. \quad (8)$$

where x are 1 bits or 0 bits. The expression (7) satisfies $V_{Rl}(n)$ and the expression (8) satisfies $V_{Rg}(n)$.

After Gray code transformation, (7) and (8) become

$$0.2^{N\text{-}1}+...+0.2^{n}+1.2^{n\text{-}1}+\mathbf{0.2^{n\text{-}2}}+x.2^{n\text{-}3}+...+x.2^{0}, \quad (9)$$

$$0.2^{N\text{-}1}+...+0.2^{n+1}+1.2^{n}+1.2^{n\text{-}1}+\mathbf{0.2^{n\text{-}2}}+...+x.2^{0} \quad (10)$$

From (9) and (10) we can see both of them always give 0 bits in ICRs $R_0(n)$ on the bit plane $n-2$.

For example, when $n = 3$, $R_0(3) = \{6, 7, 8, 9\}$. Gray codes of the decimal values from 6 to 9 have 0 bits on the bit plane 1 (= 3 − 2). This is good to optimize the probability of data bits, because the probability of 1-bit in $R_0(n)$ is always 0, and the probability of 0-bit in $R_0(n)$ is always 1.

Consequence 2.2. *In the ideal cross-points regions R_0, after Gray code transformation the entropy of obtained data on a certain bit plane is minimum.*

Proof of Consequence 2.2. This consequence is easily proved from the Definition 2.2 and Proposition 2.2. Before Gray coding, bit states around the central value 2^n in the region R_0 on the bit plane $n-2$ are random. If viewing locally in the region of cross-points $R_0(n)$, the probabilities of 1-bit and 0-bit are random, therefore the entropy of the corresponding bit string is often $H = -P(1).\log_2 P(1) - P(0).\log_2 P(0) > 0$.

After Gray code transformation, data bits in the region R_0 on the bit plane $n-2$ are all 0 bits, so the probability of 0-bit in this region is 1, and the probability of 1-bit in this region is 0, so the entropy of this bit string is always $H = -P(1).\log_2 P(1) - P(0).\log_2 P(0) = 0$, that means the average information of this region is 0.

3. THE COMBINATION OF HAAR WAVELLET AND THEORY OF IDEAL CROSS-POINTS REGIONS

The wavelet transform (WT) has gained widespread acceptance in signal processing and image compression. Recently, the JPEG committee has released its new image coding standard, JPEG-2000, which has been based upon discrete wavelet transform (DWT). We can find easily the JPEG and the JPEG-2000 in [8 - 11]. The Haar Wavelet Transform (HWT) is the efficient method in image compression and probably the simplest useful energy compression process. By using 2D Haar Wavelet Transform for input image, in wavelet image compressions parts of on image is described with reference to other parts of the same image and by doing so, the redundancy of piecewise self-similarity is exploited.

With an NxN image (.BMP), we assume A is the matrix of this image (N even). By using 2D-DHWT level 1 for A, we have the matrix B:

$$B = W_N A W_N^T \quad (11)$$

with W_N is called the Discrete Haar Wavelet Transform (DHWT).

$$W_N = \begin{bmatrix} H \\ G \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 1 & & 0 & 0 \\ \vdots & & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 \\ -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & 1 & & 0 & 0 \\ \vdots & & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix} \quad (12)$$

In the lossy compression, wavelet matrix images can be approximated to give compressed images with high compression ratios. This compression ratio still be improved by using the scheme lossless image compression with Ideal Cross-points Regions.

In Figure 1, the 2D-DWT Transform divides the image into 4 sub-bands: LL-lower resolution version of image, LH–horizontal edge data, HL–vertical edge data, HH–diagonal edge data. In every level of DHWT, the value of 3 sub-bands: [HL],[LH],[HH] can be replaced with the average value M of this area. After reodering the original input image in Figure 2(a) as an example by 2D-DHWT, it is clear from Figure 2(b) that most of the energy is contained in the top left sub-image [LL] and the least energy is in the lower right sub-image [HH]. So, the top left sub-image [LL] contents main informations of the original image. Besides, the top right sub-image [HL] contains the near-vertical edges and the lower left sub-image [LH] contains the near-horizontal edges.

In the wavelet matrix image, grey values is stabler and be focused on the top left sub-image [LL]. When the value of 3 sub-bands: [HL], [LH], [HH] are replaced by the average value M (nearly zero), it can reduce a number of bits for storing this area and make the data simpler. Thus, when we use the matrix result of the 2D-DHWT (we should use the 2D-DHWT level 1 to have the best result) to combine with the theory of ideal cross-points regions, ideal cross-points regions will be concentrated and mapped easier. The more similar the grey values in the top left sub-image [LL] is, the more convenient ideal cross-points regions dispose. The DWT is an useful step that makes the input image simpler and be arranged by blocks (sub-images). This is helpful to apply the use of the theory of ideal cross-point regions to optimize the probabilities of data bits in those regions.

After using the DHWT and approximating the wavelet matrix image, the matrix image will become stabler and simpler to compress. By using DHWT with quantization before optimizing the probability in each bit plane of ideal cross-points regions, the difference of grey values in the matrix image will lower and more similar. Thus, ideal cross-points regions will be found and mapped easier. If the step of quantization increases, the difference of data bits in the wavelet matrix will so small and the number of bit planes will decrease. As the result, the compression ratio and the distortion of output image will be higher.

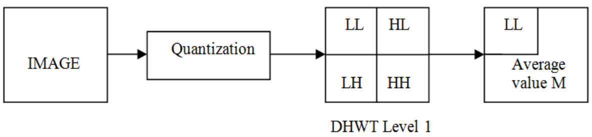

Figure 1. The Scheme of lossy 2D-DHWT level 1

(a) A (b) $B = W_N A W_N{}^T$

Figure 2. Figure 2(a) Original image and Figure 2(b) Level 1 Haar transform of Lenna image.

4. THE LOSSY CODING SCHEME FOR IMAGE USING THE HAAR WAVELLET TRANSFORM AND THE THEORY OF IDEAL CROSS-POINTS REGIONS

Figure 3 presents a broad overview of the LS-HWT&ICRs for image compression. Each step is numbered according to the sequence of the scheme, we have 9 steps from 1 to 9.

Step 1: Read the file images: this procedure will read files of images with the specific format (a bitmap file).

Step 2: Quantization: When lossy code, the accuracy of the wavelet coefficients should be chosen appropriately and the quantized coefficient is called a uniform scalar quantization. But, let us not keep an order of magnitude more or less precise coefficients.

Step 3: Haar wavelet transform: it makes the matrix image will become stabler and simpler to compress.

Step 4: We will statistic and reduce the number of gray levels in this image. This procedure will count the number of real gray levels in photographs and then arrange them. This is very benefit step because of the pixel gray levels of the transformation will tend to close together, increase the ability in all areas of cross-points has size large.

Step 5: Find and map Ideal Cross-points Regions.

Step 6: Gray Coding, this procedure will increase the ability of the same status of bits (0 or 1) around the grey value 2^n (the central value).

Step 7: Bit-planes decomposition is presented in Proposition 2.2 and Consequense 2.2. Decomposing image data into separate bit planes that are numbered from 0 to $N-1$, these numbers depend on the significance of bits, where N is the bit length of pixels of image.

Step 8: Code bit planes and compress data bits with C. B. Jones'algorithm: this procedure will code data on each bit plane from the high weight bit to the lower, from left to right and from down below. And, we can refer to [1].

Step 9: Finally, the data of compressed image will be written in a result file.

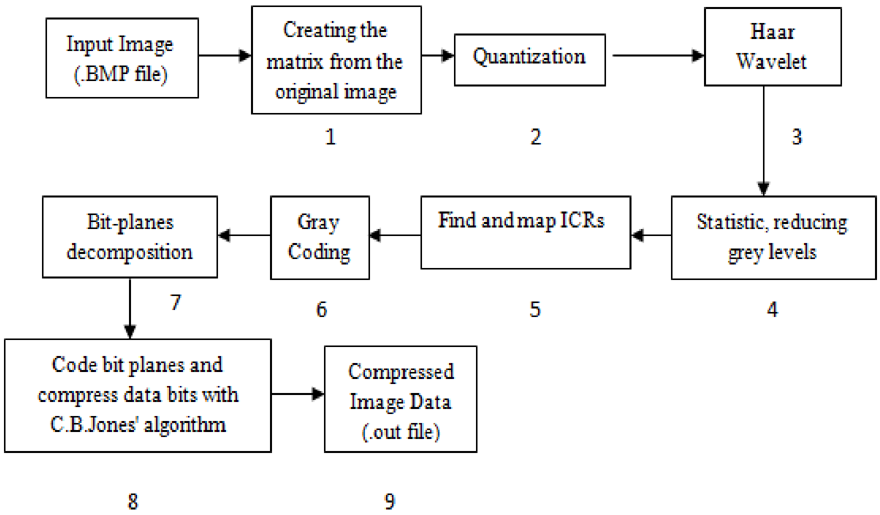

Figure 3. The Scheme of lossy image compression with DHWT and ICRs

The compressed image will be decoded with the inverse process with Figure 3.

5. EXPERIMENTAL RESULTS

Table 1 presents the results for images being compressed by LS-HWT&ICRs. The compression ratio used here is the ratio between files of images, i.e. including the headers of the original image and the compressed image. In the Table 1, the LS-HWT&ICRs and the scheme JPEG 2000 have the nearly ratio; the scheme JPEG, JPEG 2000 have the nearly value MSE, PSNR. With these results we can see that the LS-HWT&ICRs is better than the other schemes such as JPEG (DCT), JPEG 2000 (DWT) because the quality of reconstructed images is better. In the TABLE 1, the value MSE is always lower and the value PSNR is higher than these schemes.

We also have higher ratio with higher quantization steps. Certainly, the distortion of decompressed images will increase. Therefore, we must choose the suitable quantization steps to have acceptable results that also depend on types of input images.

The mean squared error (MSE) which for two $m \times n$ monochrome images I and K where one of the images is considered a noisy approximation of the other is defined as:

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \left[I(i,j) - K(i,j) \right]^2 \quad (13)$$

The peak signal-to-noise ratio (PSNR) is defined as:

$$PSNR = 10 \lg \left(\frac{MAX_I^2}{MSE} \right) \quad (14)$$

Here, MAX_I is the maximum possible pixel value of the image. When the pixels are represented using 8 bits per sample, this is 255. More generally, when samples are represented using linear Pulse-code modulation (PCM) with *B* bits per sample, MAX_I is 2^B-1.

Table 1. Experimental results of HWTICR the method

Image	.out				JPEG 2000(*)			JPEG(**)		
	MSE	PSNR	CR	Quan. Step	MSE	PSNR	CR	MSE	PSNR	CR
Zelda	93.1	28.4	7.79385	11	17852.9	5.6	7.7014	17894.4	5.6	7.6832
Moon	85.1	28.8	7.84155	10	2105.7	14.9	7.8219	2101.3	14.9	6.9223
Couple	71.9	29.6	8.80363	7	3873.3	12.2	8.1529	3842.8	12.3	6.8085
Girl	50.5	31.1	7.52807	5	6125.4	10.3	8.1984	6136.4	10.3	8.9602
Joint	11.8	37.4	9.50762	3	8932	8.6	8.2377	8895.6	8.6	18.1081
Lena	60.8	30.3	8.90397	7	5111	11.0	8.1847	5111.1	11.0	9.1459
Mandrill	373.3	22.4	8.33974	15	2953.3	13.4	8.1529	2974.4	13.4	4.4214
Chest	370.1	22.4	7.73772	13	3784.2	12.4	7.7381	3811.3	12.3	4.4828

(*)(**) are presented in the papers [8 - 11]

6. CONCLUSION

Haar Wavelet Transform offers significant improvement over previous image compression standards not only in terms of compression performance, but also in high compression ratio. The combination of the Wavelet transform and the theory of cross-points regions with Gray Code shows us an useful lossy image compression.

However, the code is still complex and the distortion factor is still high when we augment the step of the quantization. This is particularly in applications which the distortion is acceptable. Depend on the purpose and the quality of decompressed images, we will chose the scheme appropriate for compression. From this mathematically sound foundation, the problem of improving the compression ratio of image processing and transmission can be developed further in the future.

It is our hope that this paper will be able help both researchers and practitioners in applications that need optimal transformation.

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