

A NOVEL DESIGN OF THE ROOTS BLOWER

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Abstract. This paper reports a novel curve developed from non - circular gearing theory, which can be applied in rotor profile design of the two - lobe Roots blower. The formulas for calculating the volumetric efficiency and specific flow rate of the blower have also been established. To evaluate this type of the Roots blower, the volumetric efficiency and specific flow rate are being compared with those parameters of the two traditional designs and one recent variant. The results show that with the new design, the specific flow rate significantly increases for 20% to 37%, and the transverse dimension decreases for 4% to 15%. All these changes confirm usefulness and advantages of this new design.

Keywords: hydraulic machine, Roots blower, profile, flow.

Classification numbers: 5.5.1, 5.6.1, 5.10.1.

1. INTRODUCTION

Roots blower belongs to the non - contact hydraulic machines with external mating gears [1]. This kind of blowers with the rotor profile generated by circular arches was invented by the Roots brothers yet in 1860 [2]. In 1875, Palmer and Knox applied cycloidal curves in profile designing process, in which the addendum rotor was epicycloidal and the dedendum rotor was hypocycloidal [3]. Litvin in [4] simplified the rotor profile with the circular addendum and the dedendum created by the curve meshing with the circular addendum in order to guarantee continuous matching process. During the history of 150 years of research and development of the Roots blower, this type of pump has improved from the structure and design aspect, as well as been applied in more industrial fields. One of the improvement methods is to redesign the rotor profile, which can increase the blower flow without enlarging the blower size. In 2008, Hsieh and Hwang developed the blower presented in [3] by changing a trochoid rate to increase the inlet chamber and outlet chamber volumes, and by that to increase the blower flow [5]. In 2015, Hsieh proposed a novel rotor profile generated by the locus of the fixed point of ellipse rolling on the pitch circle [6]. In this paper, the optimal ratio λ between the semi - minor and semi-major axes of the rolling ellipse was proven to equal 0.6. In [7], Cai et al. used a conjugated curve in designing three - lobe rotor, in which he combined circular dedendum and cycloidal addendum (2016). In the same year, Shinde also proposed another profile with involute addendum and circular dedendum [8]. With those conjugated profiles, the specifications of the

blower, especially the flow, are distinctly improved. All these works showed us the importance of finding new and better profile in order to achieve larger flow of the blowers. In the current paper, with the purpose of increasing volumes of the inlet and outlet chambers, we propose a novel design of the Roots blower with the rotor profile $\{\Gamma\}$ generated by non-circular gearing theory. Figure 1 describes this method of profile generation: *i) the rotor addendum $\{\Gamma^d\}$ is a curve generated by point M fixed on the circle $\{\Sigma^S\}$ when $\{\Sigma^S\}$ rolling outside on the elliptical pitch $\{\Sigma^E\}$; ii) the rotor dedendum $\{\Gamma^c\}$ is a curve generated by point M fixed on the circle $\{\Sigma^S\}$ when $\{\Sigma^S\}$ rolling inside on the elliptical pitch $\{\Sigma^E\}$.*

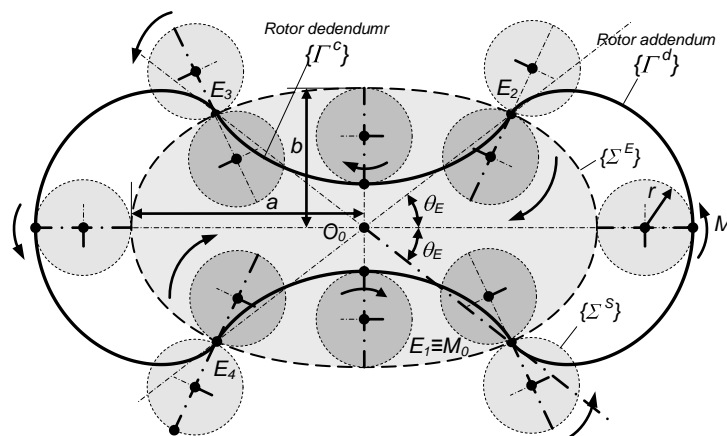


Figure 1. Principle of the generation of rotor profile.

2. MATHEMATICAL MODEL OF A NEW ROTOR FOR BLOWER

2.1. Mathematical model of the addendum rotor

To establish the equation of addendum rotor following the method of profile generation presented in section 1, in Figure 2 we have:

$\mathcal{G}_0\{O_0x_0y_0\}$: the fixed reference coordinate system with origin O_0 of $\{\Sigma^E\}$.

$\mathcal{G}_1\{O_1x_1y_1\}$: the local coordinate system with origin $O_1 \equiv P_i$ (contact point between $\{\Sigma^S\}$ and $\{\Sigma^E\}$) and axis $O_1x_1 \equiv O_0P_i$.

$\mathcal{G}_2\{O_2x_2y_2\}$: the local coordinate system connected to the rolling circle $\{\Sigma^S\}$, and with origin located at the centre of $\{\Sigma^S\}$.

φ : rotation angle of \mathcal{G}_2 in relation to \mathcal{G}_1 during relative motion of $\{\Sigma^S\}$ when rolling on $\{\Sigma^E\}$.

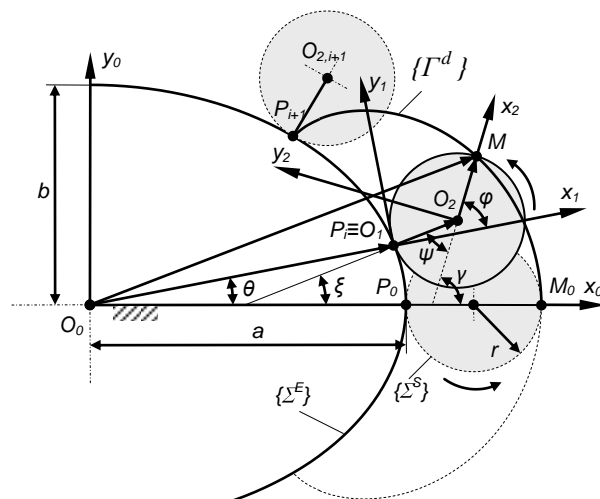


Figure 2. Principle of the generation of rotor addendum profile.

- θ : rotation angle of \mathcal{G}_1 in relation to \mathcal{G}_0 when $\{\Sigma^S\}$ rolling from P_0 (initial position) to P_i (random position).
- γ : rotation angle of \mathcal{G}_2 in relation to \mathcal{G}_0 during relative motion.
- r : radius of $\{\Sigma^S\}$.
- a, b : semi-major and semi-minor axes of $\{\Sigma^E\}$, respectively.
- ψ : angle parameter of $\{\Sigma^S\}$.

Let M be the point fixed on $\{\Sigma^S\}$, when $\{\Sigma^S\}$ is rolling outside $\{\Sigma^E\}$, from figure 2 we have:

$$\vec{r}_M = \vec{r}_{P_i} + \vec{r}_{O_2} + \vec{r}_{O_2M} \quad (1)$$

By transforming equation (1) into the algebraic form, we have:

$${}^0\underline{r}_M^d(\theta, \gamma, \psi) = {}^0\underline{r}_{P_i}(\theta) + {}^0\underline{R}_1(z, \theta) {}^1\underline{r}_{O_2} + {}^0\underline{R}_2(z, \gamma) {}^2\underline{r}_M(\psi) \quad (2)$$

where: ${}^0\underline{r}_{P_i}(\theta)$, ${}^1\underline{r}_{O_2}$, ${}^2\underline{r}_M(\psi)$ are the vectors determining positions of P_i , O_2 , M in the coordinate systems $\mathcal{G}_0, \mathcal{G}_1, \mathcal{G}_2$; ${}^0\underline{R}_1(z, \theta)$ and ${}^0\underline{R}_2(z, \gamma)$ are the rotation matrix presenting counterclockwise revolution around z axis the angles θ, γ . Developing equation (2), we have:

$$\{I^d\}: {}^0\underline{r}_M^d(\theta, \xi, \gamma) = \begin{bmatrix} x_M^d(\theta, \xi, \gamma) \\ y_M^d(\theta, \xi, \gamma) \end{bmatrix} = \begin{bmatrix} r \cos \gamma + r \cos \xi + x_P(\theta) \\ r \sin \gamma + r \sin \xi + y_P(\theta) \end{bmatrix} \quad (3)$$

equation (3) is presenting profile of the rotor addendum. This equation contains three parameters γ, ξ, θ , and it is necessary to find the relation between them.

• **Determining** $\xi = \xi(\theta)$

From figure 2, it is clearly shown that ξ is the angle between the common normal of $\{\Sigma^E\}$ and $\{\Sigma^S\}$ on the contact point P_i and the axis O_0x_0 . Therefore ξ is given by:

$$\xi(\theta) = \tan^{-1} \left(\frac{-\partial x_P(\theta) / \partial \theta}{\partial y_P(\theta) / \partial \theta} \right) \quad (4)$$

where:

$$\begin{bmatrix} x_P(\theta) \\ y_P(\theta) \end{bmatrix}^T = \begin{bmatrix} \rho_P(\theta) \cos \theta \\ \rho_P(\theta) \sin \theta \end{bmatrix}^T \quad (5)$$

with $\rho_P(\theta)$ is a distance from arbitrary point P_i on $\{\Sigma^E\}$ to the origin O_0 of \mathcal{G}_0 . According to [9], $\rho_P(\theta)$ is expressed by:

$$\rho_P(\theta) = \frac{2ab}{a+b-(a-b)\cos(2\theta)} \quad (6)$$

• **Determining** $\gamma = \gamma(\theta)$

From 2, using relation between the angles of triangle, therefore $\gamma(\theta)$ is presented by:

$$\gamma(\theta) = \xi(\theta) + \psi(\theta) \quad (7)$$

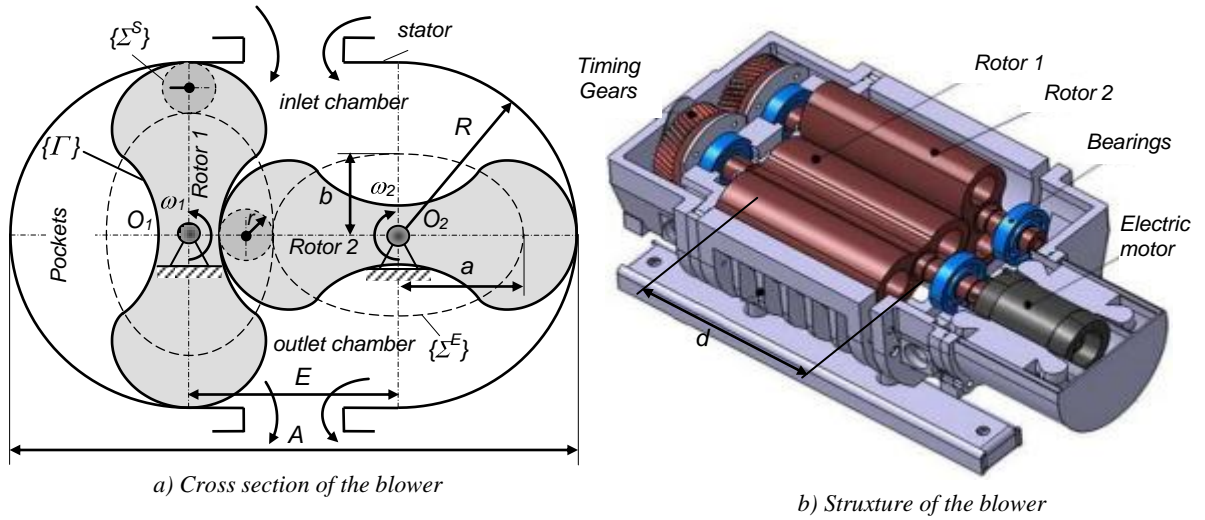


Figure 4. Principle of blower design.

Let: E , R , A , d be distance of the shafts, radial dimension, transverse dimension of the blower, axial dimension of the inlet and outlet chamber, from figure 4 we have:

$$E = a + b \quad (12)$$

$$R = a + 2r \quad (13)$$

$$A = E + 2R \quad (14)$$

Therefore, parameters E , R , A , d are the design parameters of the blower, and a , b , r are the characteristic design parameters of the rotor as well as the blower itself.

2.3. Condition of the characteristic design parameters for generation profile of the blower

There is a problem that not every set of the characteristic design parameters a , b , r can help to generate the rotor profile. Practically, the rotor profile can not be generated by some sets of parameters, and interference between profiles or undercutting phenomenon can happen with the other sets. It raises the need to set condition for those parameters. Let C^E , C^S be circumference of $\{\Sigma^E\}$ và $\{\Sigma^S\}$, respectively:

$$C^E = \int_0^{2\pi} \left[\left(\frac{\partial x_P(\theta)}{\partial \theta} \right)^2 + \left(\frac{\partial y_P(\theta)}{\partial \theta} \right)^2 \right]^{\frac{1}{2}} d\theta \quad (15)$$

$$C^S = 2\pi r \quad (16)$$

following the principle of profile generation presented in section 2, $\{\Sigma^S\}$ is only rolling on $\{\Sigma^E\}$. The symmetry of the rotor profile is also taken into consideration, we have:

$$C^E = 4C^S \quad \text{or} \quad \int_0^{2\pi} \left[\left(\frac{\partial x_P(\theta)}{\partial \theta} \right)^2 + \left(\frac{\partial y_P(\theta)}{\partial \theta} \right)^2 \right]^{\frac{1}{2}} d\theta = 8\pi r \quad (17)$$

on the other hand, from figures 1 and 4, if the profile is symmetrical and satisfies (17), then radius of the rolling circle $\{\Sigma^S\}$ need to fulfil following condition:

$$r < \frac{b}{2} \tag{18}$$

Let $\lambda = b/a$ be the characteristic design parameter of the blower. By substituting λ into (17) and (18), we have:

$$0.5 \leq \lambda \leq 1 \tag{19}$$

Formulas (18) và (19) are boundary condition of the characteristic design parameters for generating the rotor profile of the blower.

3. DESIGN AND EVALUATION

3.1. Theoretical specific flow rate

Specific flow rate of blower is defined by the volume amount discharged out while the driving shaft accomplish one revolution. According to [10], the specific flow rate Q_r is given by:

$$Q_r = 2ZSd \tag{20}$$

where: d is the radial dimension of the blower; Z is the number of the rotor teeth; S is the area of perpendicular cross section (see Figure 5). From Figure 5, we have:

$$S = \frac{1}{2} \left[\pi R^2 - S_{\Gamma}^- \right] \tag{21}$$

with S_{Γ}^- is the area of the rotor cross section perpendicular to the blower shaft, and S_{Γ}^- is given by:

$$S_{\Gamma}^- = 4(S_{\Gamma}^d + S_{\Gamma}^c) \tag{22}$$

where S_{Γ}^d and S_{Γ}^c are the areas of the sub cross - section bordered by the profiles of addendum rotor and dedendum rotor in relation to the origin O_2 (figure 5). S_{Γ}^d and S_{Γ}^c are calculated as follows:

$$S_{\Gamma}^d = \int_0^{\theta_E} - \frac{\partial x_M^d(\theta, \xi, \gamma)}{\partial \theta} y_{\Gamma}^d(\theta, \xi, \gamma) d\theta \tag{23}$$

$$S_{\Gamma}^c = \int_{\theta_E}^{\pi/2} - \frac{\partial x_M^c(\theta, \xi, \gamma)}{\partial \theta} y_{\Gamma}^c(\theta, \xi, \gamma) d\theta. \tag{24}$$

Case study

Given are the set of characteristic design parameters: radius of $\{\Sigma^S\}$ $r = 11.6704mm$; rolling ellipse $\{\Sigma^E\}$ with major semiaxis $a = 56.6591mm$, minor semiaxis $b = 33.9955mm$; radial

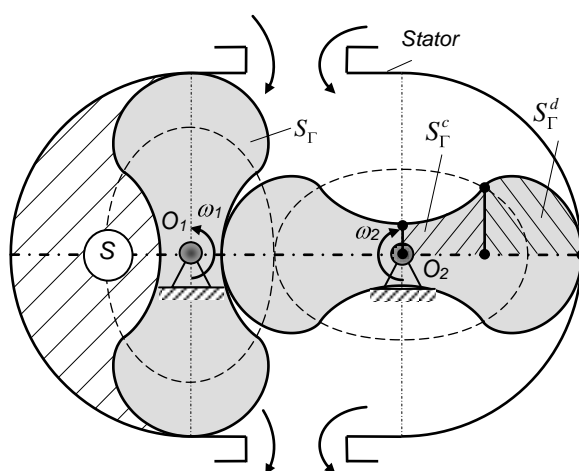


Figure 5. Calculation of the specific flow rate.

dimension $d = 130\text{mm}$; number of rotor teeth $Z = 2$. The specific flow rate of the blower will be equal $Q_r = 3.37 \times 10^6 \text{ mm}^3/\text{rev}$.

3.2. Volumetric efficiency of the blower

According to [6], to access the theoretical flow of the blower, the volumetric efficiency η is being used. This parameter is defined by the ratio between the volume blown from inlet to outlet of the blower in one revolution of the driving shaft and the total volume measured inside of the blower stator. From Figure 5, we have:

$$\eta = \frac{2ZS}{S_{stator}} 100\% \quad (25)$$

where: S_{stator} is the area of the inside blower chamber (See figure 4), and S_{stator} is given by:

$$S_{stator} = (a + 2r) \left[r(a + 2r) + 2(a + b) \right] \quad (26)$$

3.3. Evaluation

To prove the advantages of this design, the authors have carried out comparison of the volumetric efficiency, specific flow rate and blower dimension between the proposed design and two traditional variants - type 1 [3], type 2 [4], which were presented in [6, 11, 12] (Those two traditional designs had already been applied in manufacturing real product). A new design of Hsieh [6] - called type 3 is also added to comparing process.

a) Type 1: Traditional design (Palmer and Knox [3] in 1875)

- *Profile equation:* According to [3], the rotor profile $\{\Gamma\}$ consist of design epicycloidal addendum $\{\Gamma^d\}$ and hypocycloidal dedendum $\{\Gamma^c\}$:

$$\{\Gamma\}: r_{\Gamma} = \begin{bmatrix} \mp r_s \cos \frac{R_L \pm r}{r_s} \theta + (R_L \pm r_s) \cos \theta \\ -r_s \sin \frac{R_L \pm r_s}{r_s} \theta + (R_L \pm r_s) \sin \theta \end{bmatrix} \quad (27)$$

where: R_L, r, θ are radius of the circle pitch $\{\Sigma^L\}$, radius of the rolling circle $\{\Sigma^S\}$ and parametric angle $\{\Sigma^L\}$, respectively (Figure 6). The signs “ \pm ” and “ \mp ” are chosen by the following rules: the upper sign for $\{\Gamma^d\}$, the lower sign for $\{\Gamma^c\}$.

- *Dimensional design parameters:* from figure 6, those parameters are given by:

$$\begin{cases} E = 2R_L \\ R = R_L + 2r \\ A = E + 2R \end{cases} \quad (28)$$

- *Condition for profile generation:* to generate the profile, according to [10], the characteristic design parameters need to satisfy:

$$R_L = 2Zr \quad (29)$$

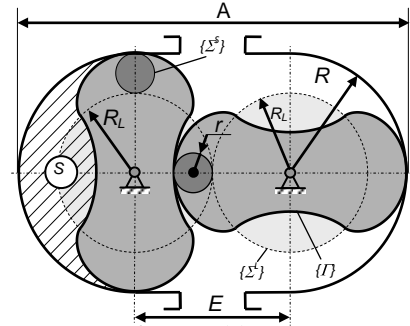


Figure 6. Roots blower type 1.

b) Type 2: Traditional design (Litvin [4] in 1956)

- *Profile equation:* In [4], the rotor addendum profile $\{\Gamma^d\}$ are drawn by circular arch (Figure 7), with equation:

$$\{\Gamma^d\}: r_{\Gamma^d} = \begin{bmatrix} \rho \cos \theta + c \\ \rho \sin \theta \end{bmatrix} \quad (30)$$

The rotor dedendum profile $\{\Gamma^c\}$ is the meshing curve with $\{\Gamma^d\}$ (Figure 7) presented by following equation:

$$\{\Gamma^c\}: r_{\Gamma^c} = \begin{bmatrix} \rho \cos(\theta - 2\alpha) + c \cos 2\alpha - 2R_L \cos \alpha \\ \rho \sin(\theta - 2\alpha) - c \sin 2\alpha + 2R_L \sin \alpha \end{bmatrix} \quad (31)$$

where: R_L , c , ρ are radius of $\{\Sigma^L\}$, distance from origin O_1 to the centre of the rotor circular arch, radius of the rotor addendum $\{\Gamma^d\}$, respectively. And θ, α are the parametric angle of $\{\Sigma^L\}$ and rotation angle of the driving shaft.

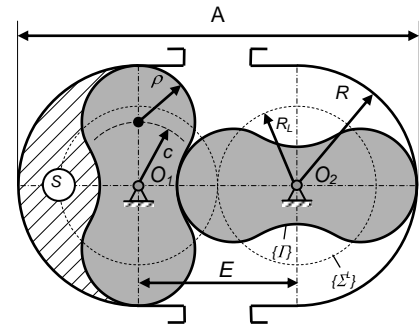


Figure 7. Roots blower type 2.

- *Dimensional design parameters:* from Figure 7, the design parameters of the blower are:

$$\begin{cases} E = 2R_L \\ R = c + \rho \\ A = E + 2R \end{cases} \quad (32)$$

- *Condition for profile generation:* According to [4], $\lambda = c/R_L$ is the *characteristic design parameter* of the rotor profile. Condition for profile generation with no singularities is:

$$0.5 \leq \lambda < 0.9288 \quad (33)$$

c) Type 3: new design in 2015 (Hsieh [6])

- *Profile equation:* In this type of blower, $\{\Gamma^d\}$ is the locus of a point fixed on the rolling ellipse $\{\Sigma^{ES}\}$, when $\{\Sigma^{ES}\}$ is only rolling outside on $\{\Sigma^L\}$ of the gear. $\{\Gamma^c\}$ is the locus of a point fixed on $\{\Sigma^{ES}\}$, when $\{\Sigma^{ES}\}$ is only rolling inside on $\{\Sigma^L\}$ (Figure 8). The equation of $\{\Gamma\}$ is given by:

$$\{\Gamma\}: r_{\Gamma} = \begin{bmatrix} \mp a_1(1 - \cos \psi) \cos(\xi \pm \theta) \pm b_1 \sin(\xi \pm \theta) \sin \psi + R_L \cos \theta \\ -a_1(1 - \cos \psi) \sin(\xi \pm \theta) - b_1 \cos(\xi \pm \theta) \sin \psi + R_L \sin \theta \end{bmatrix} \quad (34)$$

where: R_L, a_1, b_1 are radius of $\{\Sigma^L\}$, major semi-axis of $\{\Sigma^{ES}\}$ and minor semi-axis of $\{\Sigma^{ES}\}$, respectively. And θ, ψ are the parametric angle of $\{\Sigma^L\}$ and $\{\Sigma^{ES}\}$. ξ is the angle between the common normal of $\{\Sigma^L\}$ and $\{\Sigma^{ES}\}$ on the contact point, and the line connecting the centers of the rotor 1 and rotor 2.

- *Dimensional design parameters:* from Figure 8, those parameters are presented as below:

$$\begin{cases} E = 2R_L \\ R = R_L + 2a_1 \\ A = E + 2R \end{cases} \quad (35)$$

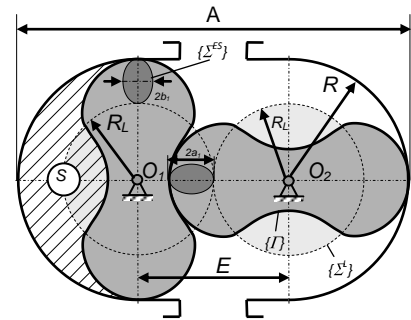


Figure 8. Roots blower type 3.

- *Condition for profile generation:* As stated in [6], the characteristic design parameter is calculated by formula $\lambda = a_1/b_1$. Condition for profile generation with no self-intersection:

$$0.4 \leq \lambda \leq 1. \quad (36)$$

d) Analysis and evaluation

Let take two case studies into consideration:

- **Case study 1:** Determine characteristic design parameters of the variants of the blower, of which the radial dimension $R = 72$ mm and the axial dimension $d = 150$ mm. The characteristic parameters are shown in Table 1.

Table 1. Characteristic design parameters.

Blower type	Characteristic design parameters		A [mm]	Volumetric efficiency
	Parameters	Value		
Type 1 (Fig. 6) (1875)	R_L [mm]	48.0000	240.0000	$\eta = 54.09\%$
	r [mm]	12.0000		
Type 2 (Fig. 7) (1956)	R_L [mm]	44.0921	232.1842	$\eta = 63.66\%$
	c [mm]	39.6829		
	ρ [mm]	32.3171		
Type 3 (Fig. 8) (2015)	R_L [mm]	43.6754	231.3508	$\eta = 64.49\%$
	a_l [mm]	14.1623		
	b_l [mm]	7.0811		
Roots blower with new design (Fig. 5)	a [mm]	51.6393	221.4587	$\eta = 81.21\%$
	b [mm]	25.8196		
	r [mm]	10.1803		

From Table 1 we have:

- The volumetric efficiency is increasing, and the transverse dimension is decreasing in the history of blower development, which matches well with our notice made in section 1.
 - The volumetric efficiency of the newly proposed blower is distinctly higher than efficiency of the older designs: 27.12 % higher in comparison with type 1, 18.15 % higher than type 2, and 16.72 % higher than type 3. It means that the specific flow rate is the largest one, while dimension of the novel blower is smallest, which leads to consideration that the novel design is approximately optimal. For thorough evaluation we are going to examine case number 2 below.
- **Case study 2:** In order to generally evaluate, we are going to determine the characteristic design parameter of each type of blower based on parameter λ (λ needs to satisfy conditions 19, 29, 33, 36 for each type of blower to generate profile), with constraint of radial dimension $R = 72$ mm and axial dimension $d = 150$ mm for all of the evaluated blowers. On the other hand, from conditions (19 and 36), when $\lambda = 1$, both $\{\Sigma^S\}$ and $\{\Sigma^{ES}\}$ become a circle, which means that blower of type 3 and actually proposed blower will transform to type 1. Therefore, if the increment $\Delta\lambda = 0.1$ to satisfy conditions (19, 29, 33, 36), then $\lambda \in [0.5 \div 1]$. From the values of λ , the relation of the characteristic design parameters for each type will be found. By

substitution to formulas (12, 13, 17, 29, 32, 23), and solve this repeating problem in Matlab, the characteristic design parameters will be listed in the Table 2 below.

Table 2. Characteristic design parameters of each type of blower based on parameter λ .

Blower type	Characteristic design parameters	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$	$\lambda = 0.8$	$\lambda = 0.9$	$\lambda = 1.0$
Type 2 (1956)	R_L [mm]	63.0654	59.3078	55.4314	51.5470	47.7664	-
	c [mm]	31.5327	35.5847	38.8020	41.2376	42.9898	-
	ρ [mm]	46.4673	42.4153	39.1980	36.7624	35.0102	-
Type 3 (2015)	R_L [mm]	47.3150	48.2868	49.2524	50.1980	51.1156	52.0000
	a_l [mm]	15.3425	14.8566	14.3738	13.9010	13.4422	13.0000
	b_l [mm]	7.6713	8.9139	10.0617	11.1208	12.0979	13.0000
Roots blower with new design	a [mm]	55.9426	55.2426	54.4895	53.6912	52.8578	52.0000
	b [mm]	27.9713	33.1456	38.1426	42.9529	47.5720	52.0000
	r [mm]	11.0287	11.3787	11.7552	12.1544	12.5711	13.0000

Note: when $\lambda = 1$ blower of type 3 and new design of blower will transform to type 1.

Applying equations (14, 28, 32, 35) for the data calculated in Table 2, we can obtain the graph in Figure 9.

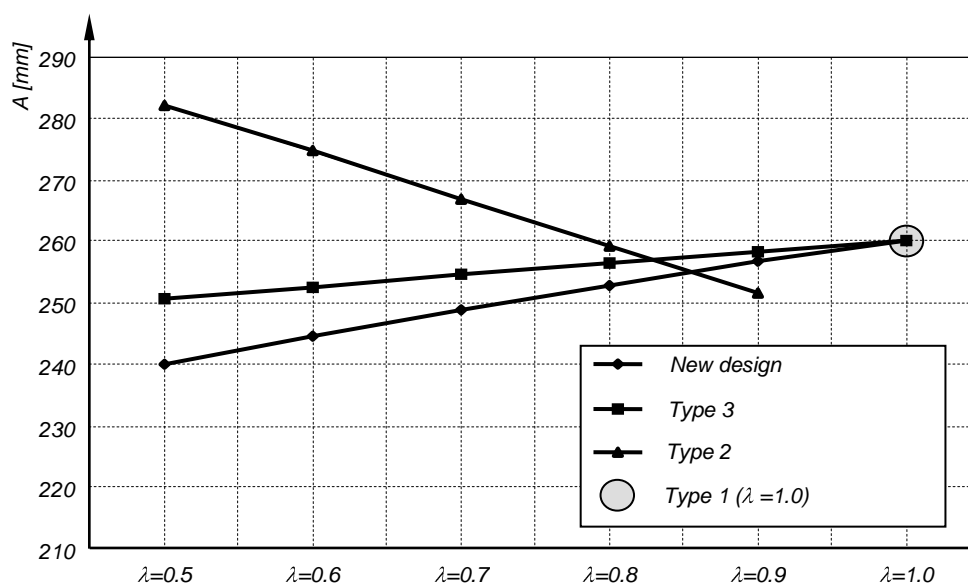


Figure 9. Transverse dimension of the blower types.

From the graph 9, it is noticeable that the transverse dimension A of the novel design is the smallest. And together with type 3, this dimension will increase and reach maximum value when $\lambda = 1.0$ (become type 1). On the other hand, the blower of type 2 will have decreasing transverse dimension when λ increases. By applying the new design, the dimension of blower will be smaller for about 4 % to 15 % in comparison with the previous types.

To evaluate the flow of the new design, by substitution the characteristic design parameters in Table 2 into the profile equation $\{ \Gamma \}$ given by formulas (10, 22, 25, 26, 29), and by calculation similarly in section 3.1 with S is the area of the cross sections described in figures 5, 6, 7, and 8, corresponding with each type of blowers, we can obtain graph 10 below.

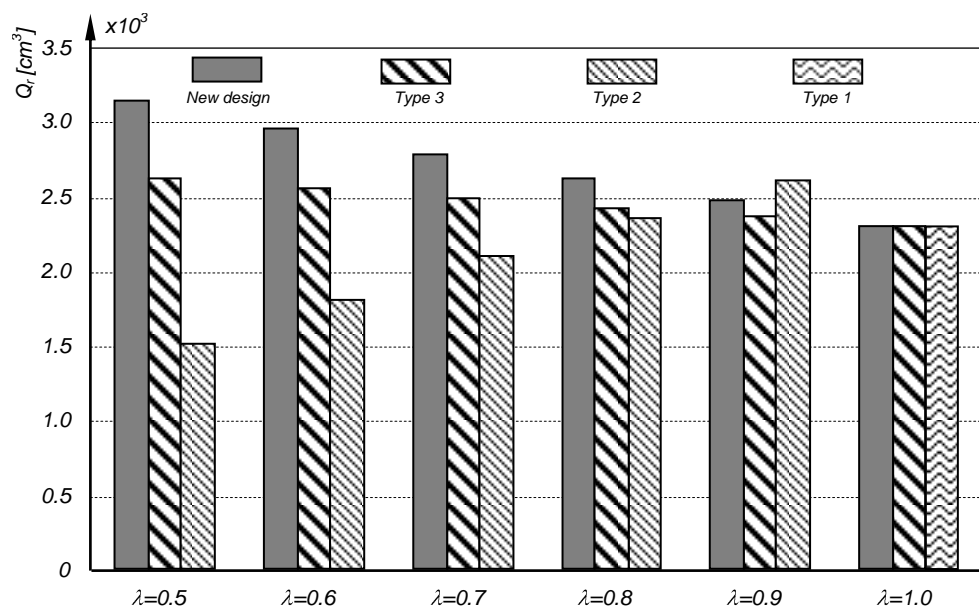


Figure 10. Flow rates of the blower types.

From Figure 10, when λ is increasing, we have:

- (i) The specific flow rate of the proposed blower and type 3 decreases when value λ rises and reaches minimum value when these two type transform to type 1 ($\lambda = 1.0$). However, when ($\lambda = 0.5$), the specific flow rate of the proposed blower is 20 % larger than the flow rate of the blower type 3, and 37% larger in comparison with type 1.
- (ii) The theoretical specific flow rate of the blower type 2 increases when λ rises from 0.5 to 1, and reaches maximum value with $\lambda = 0.9$. However, it is still lower for 21 % than the flow rate of the new design.

Finally, it is clearly shown that with the blowers of type 2, type 3 and of new design, when the transverse dimension A increases, the flow rate will decrease and vice versa. Only with the blowers of type 1, in order to achieve larger flow rate, the dimension of the blower also need to be raised. All of these have confirmed the advantages of the newer types of blower.

4. CONCLUSIONS

This research has proposed the novel curve developed from non - circular gearing theory, i.e. elliptical gears, which can be applied in designing Roots blower. In comparison of the new

designed blower with the previous ones, the volumetric efficiency and theoretical specific flow rate (in one revolution) increase from 20 % to 37 % (*with the same radial and axial dimension*), and transverse dimension decreases from 4 % to 15 %. All of these changes confirm the advantages of the proposed blower, from size and flow aspects. Therefore, this new variant can be useful when designing industrial Roots blower, when only flow rate is taken into consideration. Some applications can be listed as fruit and vegetable driers, oxygen provider for the oven of the thermal power plants etc. But it stills needs to be improved when stability of the flow rate as well as stability of the pressure are required.

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